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TRENDS AND LOCAL EFFECTS IN AVIATION ACCIDENT RATES RELATED TO DEREGULATION

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Abstract

When analyzing flight accident data over some period of time, it is clear that the rates of serious accidents per year show a steady decline. For a recent analysis of this see e.g., Landsberg[2]. However, by focusing only on long term trends it is easy to overlook local effects like sudden drops or increases in the accident rates. When using standard statistical methods like regression analysis, local effects have a tendency to be reduced to a few scattered outliers. As a result important issues affecting the accident rates may not be addressed. In this paper we shall study the accident rates for general aviation in USA for the period 1960 - 2003. In particular we will focus on a special period in the years around 1980 when the aviation business was deregulated. We will show that during this short period the accident rates were significantly lower than one could expect.

1 Introduction

When analyzing flight accident data over some period of time, it is clear that the rates of serious accidents per year show a steady decline. For a recent analysis of this see e.g., Landsberg[2]. In the present paper we shall study the accident rates for general aviation in USA for the period 1960 - 2003 with special emphasis on the period around the US. deregulation, i.e., the period around 1980. The data used here, is obtained from AOPA[1], with the US FAA (Federal Aviation Authorities) as source. A plot of the total number of accidents (both fatal and nonfatal) per 100,000 flight hours is presented in Figure 1, while the corresponding results for fatal accidents are given in Figure 2.

By considering the plots we see that rates show a steady decline. For the fatal accident rates, the trend appears to be almost linear, while the total accident rates appears to flatten out more and more. In pure numbers the fatal accident rate is reduced from 3.27 in 1960 to 1.36 in 2003, i.e., more than 50 % decrease. For the accident rates the results are even more dramatic with a reduction from 36.50 in 1960 to just 6.71 in 2003.

In addition to the long term trends the accident rate curves also have some local deviations from the general trends. In particular we observe that there is a noticeable drop in the rates around year 1980. A natural question to ask is whether this drop is statistically significant, or if this is just some random noise.

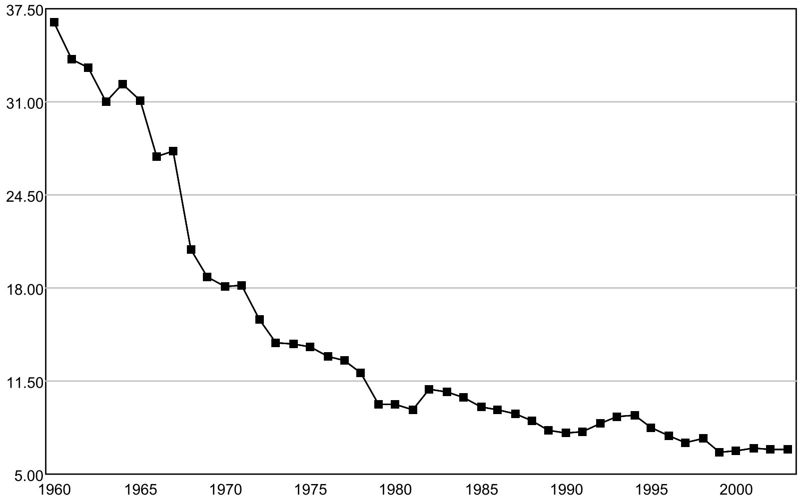

Figure 1: *Total Accident Rates 1960 - 2003*

In Section 2 we will analyze this from a "global" perspective using a standard regression model combined with an outlier test. In Section 3 we take a closer look at this phenomenon by introducing a "local" model.

2 Global Trend Analysis

Before we present the results of the analysis, we introduce the basic model. In the analysis we have included data for the years 1960 to 2003, i.e., for $n = 44$ years. We refer to the ith year as y_i, $i = 1, \ldots, n$. Thus, $y_1 = 1960$, $y_2 = 1961$, and so forth up to $y_n = y_{44} = 2003$. For each year we have recorded the number of hours flown, the number of accidents, and the number of fatal accidents. Thus, we introduce for $i = 1, \ldots, n$:

$$\begin{aligned} t_i &= \text{Number of 100,000 hours flown in year } i, \\ X_i &= \text{Number of accidents in year } i, \\ Z_i &= \text{Number of fatal accidents in year } i, \end{aligned} \tag{2.1}$$

In this setting we consider the t_is to be given constants, while the X_is and the Z_is are stochastic variables. A natural model for the stochastic variables is the Poisson model. Thus, we assume that all the stochastic variables are independent and that for $i = 1, \ldots, n$:

$$\begin{aligned} X_i &\sim \mathcal{P}o(\lambda_i t_i), \\ Z_i &\sim \mathcal{P}o(\mu_i t_i), \end{aligned} \tag{2.2}$$

where λ_i is the accident rate (per 100,000 hours flown) in year i, while μ_i is the fatal accident rate (per 100,000 hours flown) in year i, $i = 1, \ldots, n$. From this

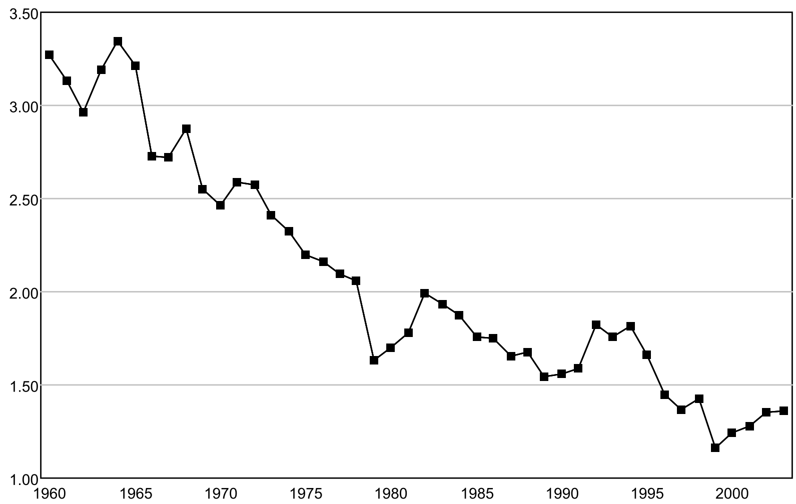

Figure 2: *Fatal Accident Rates 1960 - 2003*

it follows that the mean and the standard deviations of the variables are:

$$\begin{aligned} E[X_i] &= \lambda_i t_i, \\ SD[X_i] &= \sqrt{\lambda_i t_i}, \\ E[Z_i] &= \mu_i t_i, \\ SD[Z_i] &= \sqrt{\mu_i t_i}, \end{aligned} \tag{2.3}$$

for $i = 1, \ldots, n$. Hence, unbiased estimators for these rates are:

$$\begin{aligned} \hat{\lambda}_i &= \frac{X_i}{t_i},\ i = 1, \ldots, n, \\ \hat{\mu}_i &= \frac{Z_i}{t_i},\ i = 1, \ldots, n. \end{aligned} \tag{2.4}$$

The plotted rates in Figure 1 and Figure 2 are derived using (2.4).

When the expected value of a Poisson variable is large (larger than 100), this distribution can be approximated very accurately by the Gaussian distribution. In our case all observed values of the X_is and Z_is are greater than 300, indicating that the Gaussian distribution should fit very nicely. Thus, we have for $i = 1, \ldots, n$, that:

$$\begin{aligned} X_i &\approx \mathcal{N}(\lambda_i t_i, \sqrt{\lambda_i t_i}), \\ Z_i &\approx \mathcal{N}(\mu_i t_i, \sqrt{\mu_i t_i}), \end{aligned} \tag{2.5}$$

or equivalently:

$$\begin{aligned} \frac{X_i}{t_i} &\approx \mathcal{N}(\lambda_i, \sqrt{\lambda_i / t_i}), \\ \frac{Z_i}{t_i} &\approx \mathcal{N}(\mu_i, \sqrt{\mu_i / t_i}). \end{aligned} \tag{2.6}$$

We will also need similar results for logarithmic values. Thus, we consider the quantities $\ln(X_i/t_i)$ and $\ln(Z_i/t_i)$. By Taylor expansion around the mean values, we get that:

$$\begin{aligned}
\ln(\frac{X_i}{t_i}) &\approx \ln(\lambda_i) + \frac{X_i/t_i - \lambda_i}{\lambda_i} = \frac{X_i}{\lambda_i t_i} - 1 + \ln(\lambda_i), \\
\ln(\frac{Z_i}{t_i}) &\approx \ln(\mu_i) + \frac{Z_i/t_i - \mu_i}{\mu_i} = \frac{Z_i}{\mu_i t_i} - 1 + \ln(\mu_i).
\end{aligned} \tag{2.7}$$

From this it follows that:

$$\begin{aligned}
\ln(X_i/t_i) &\approx \mathcal{N}(\ln(\lambda_i), 1/\sqrt{\lambda_i t_i}), \\
\ln(Z_i/t_i) &\approx \mathcal{N}(\ln(\mu_i), 1/\sqrt{\mu_i t_i}),
\end{aligned} \tag{2.8}$$

for $i = 1, \ldots, n$.

Armed with these asymptotic results we then formulate our trend models. The first model is a simple linear regression model where:

$$\begin{aligned}
\lambda_i &= \alpha_1 + \beta_1 y_i, \\
\mu_i &= \alpha_2 + \beta_2 y_i,
\end{aligned} \tag{2.9}$$

for $i = 1, \ldots, n$. Using a standard least square approach, we obtain estimates for the regression parameters given in Table 1.

Parameter	Estimate
α_1	1230.19
β_1	-0.61352
α_2	93.04
β_2	-0.04591

Table 1: Estimated Linear Regression Parameters

In order to evaluate these models we use the above asymptotic results. A special feature with these models is that both the means the standard deviations are functions of the regression parameters. This property should ideally be taken into account when computing P-values etc. It turns out, however, as often is the case in such models, that if this is done, the observed deviations from the regression model are much larger than one could expect from the stochastic model. This phenomenon is known as *overdispersion*. A simple way around this problem is to use nonparametric estimators (i.e., estimators that do not use the specific Poisson relation between the mean and the standard deviation) for the standard deviations in the model. This is sometimes referred to as a quasi-Poisson model. This way, one essentially ends up with a standard regression analysis model with Gaussian errors. We will use this approach in the following.

In Figure 3 we have plotted the results of the regression analysis for the accident rates, while Figure 4 contains the corresponding results for the fatal accident rates. Included in the plots are the observed rates, the fitted regression line as well as a 95% prediction interval around the regression line.

Considering Figure 3 we see that almost all the observed values fall within the prediction interval. Thus, the results indicate that no outliers are present in

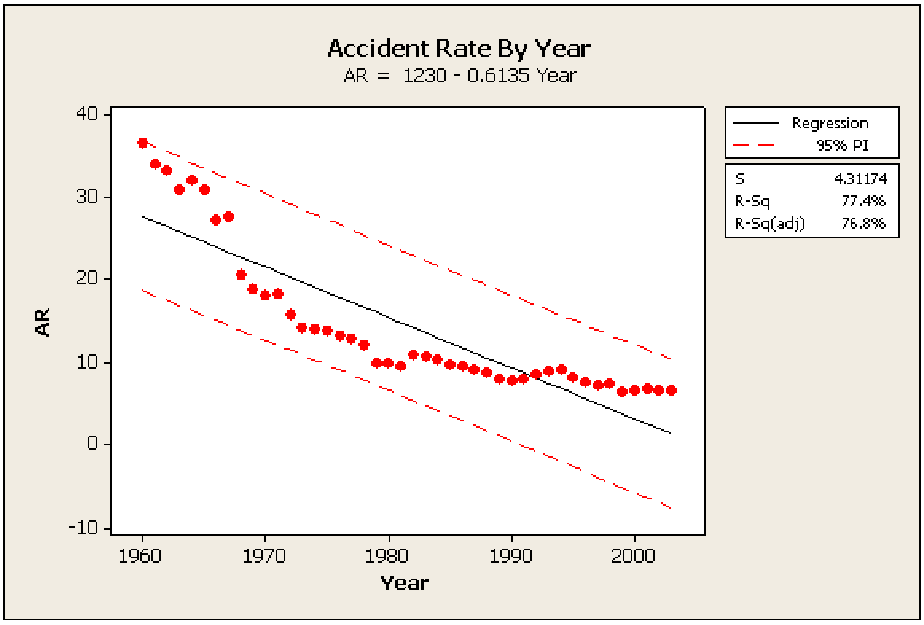

Figure 3: *Total Accident Rates 1960 - 2003*

the data. However, we also note that the linear regression line does not fit the data very well. In the early and late years the observations are systematically above the regression line, while in the intermediate years the observations are systematically below this line. This suggest that a loglinear model might be better.

Considering Figure 4 we see that except for a few points the observed values fall within the prediction interval. In this case the regression line appears to fit the data satisfactory. Still it may be of interest to consider a loglinear option as well.

In order to take a closer look at how well the model fits the data, we have included normal plots of the residuals. See Figure 7 and Figure 8 in the Appendix. In a Normal plot the y-axis is scaled so that the Gaussian cumulative distribution curve becomes a straight line. The dots in the plot represent the empirical cumulative distribution function of the standardized residuals. Thus, ideally the dots should lie close to the straight line representing a perfect Gaussian cumulative distribution curve. We observe that for the Total Accident Rates there appears to be some systematic deviations from the Gaussian curve. This mostly due to the lack of linearity in the data. For the Fatal Accident Rates the fit is acceptable except for the two negative values located in the left end of the scale. These dots corresponds to the observed outliers.

In order to improve the model fits we now consider a loglinear regression model:

$$\begin{aligned} \ln(\lambda_i) &= \alpha_3 + \beta_3 y_i, \\ \ln(\mu_i) &= \alpha_4 + \beta_4 y_i, \end{aligned} \tag{2.10}$$

for $i = 1, \ldots, n$. Again, using a standard least square approach, we obtain estimates for the regression parameters given in Table 2.

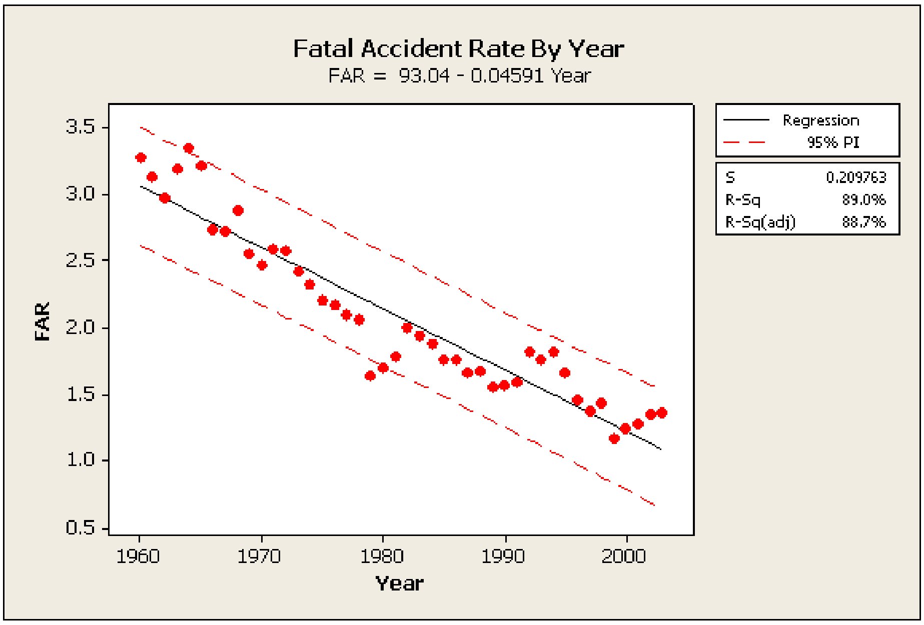

Figure 4: *Fatal Accident Rates 1960 - 2003*

Parameter	Estimate
α_3	80.73
β_3	-0.03947
α_4	44.34
β_4	-0.02203

Table 2: Estimated Loglinear Regression Parameters

In Figure 5 we have plotted the results of the regression analysis for the accident rates, while Figure 6 contains the corresponding results for the fatal accident rates. Included in the plots are the observed rates, the fitted regression line as well as a 95% prediction interval around the regression line.

Considering the total accident rates we see that the loglinear model plotted in Figure 5 appears to be a much better fit compared to the linear model in Figure 3. Still there seems to be some systematic deviations from the model as the observed values appear to flatten out faster than the fitted regression line. Moreover, as we saw in the linear case almost all the observed values fall within the prediction interval. Thus, the results indicate that no outliers are present in the data.

Considering the fatal accident rates it is not easy to tell which of the two models, Figure 4 or Figure 6, which is the best one. However, by considering the R^2 statistics which represent the amount of uncertainty explained by the models, we see that the loglinear model ($R^2 = 91.6\%$) is slightly better than the linear model ($R^2 = 89.0\%$). Finally, except for a few points the observed values fall within the prediction interval.

As we did for the linear models, we have included normal plots of the residuals. See Figure 9 and Figure 10 in the Appendix. The lack of linearity in the total accident data shows up in the normal plot as it did in the linear case,

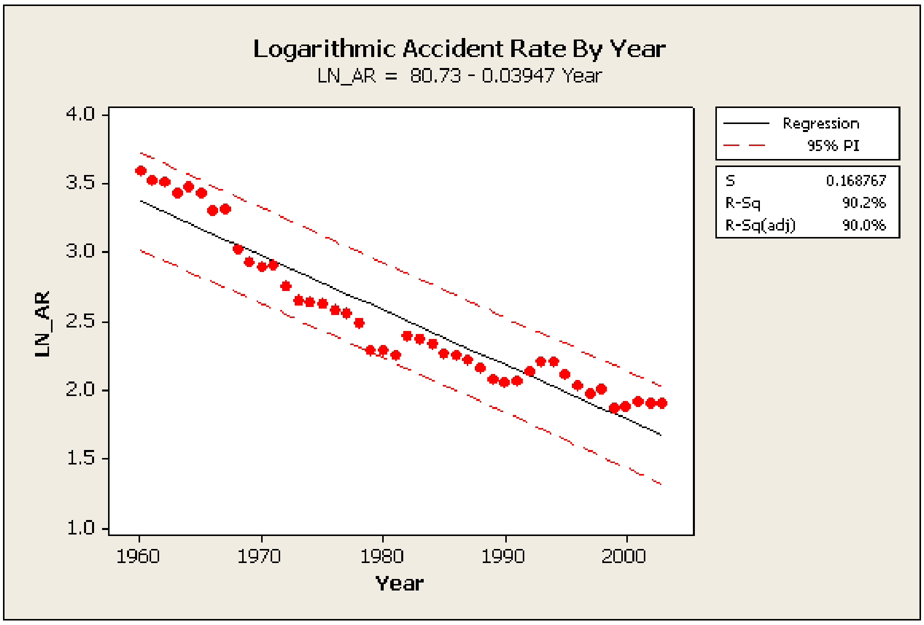

Figure 5: *Logarithmic Total Accident Rates 1960 - 2003*

although the deviations are somewhat less severe here. For the fatal accident data the normal plot is similar to the linear case. However, the leftmost points are now even further away from the straight line. This again suggests that these points are outliers. In the next section of the paper we will focus on these points.

3 Local Drop Analysis

In this section we shall take a closer look at the period around the US. deregulation, i.e., the period around 1980. During the global trend analysis we observed that there is a noticeable drop in the fatal accident rates around year 1980. So now we ask if this drop is statistically significant, or just some random noise.

In order to investigate this we start out by considering the two years when the drop occurs, i.e., 1978 and 1979. For such a short period of time, we may ignore the trend effect, and assume that the fatal accident rates are approximately equal. Since, however, the common fatal accident rate is unknown, we may include this uncertainty into the model. We have chosen to do so by using a Bayesian approach where we model the uncertainty about the fatal accident rate in terms of a prior. More specifically, we denote the common fatal accident rate for 1978 and 1979 by μ and assume that:

$$\mu \sim \mathcal{G}amma(a, b), \tag{3.1}$$

where a and b are chosen so that $E[\mu] = a/b = 1.8470$ which is the average value of the observed fatal accident rates in 1978 and 1979. In addition to this, we want to minimize the effect of the prior on the results. This is achieved by choosing "small" numbers for a and b. This leads to the following values: $a = 1.0$ and $b = 0.5414$. We then turn to the variables Z_{19} and Z_{20}, representing the numbers of fatal accidents in the years 1978 and 1979 respectively. Given

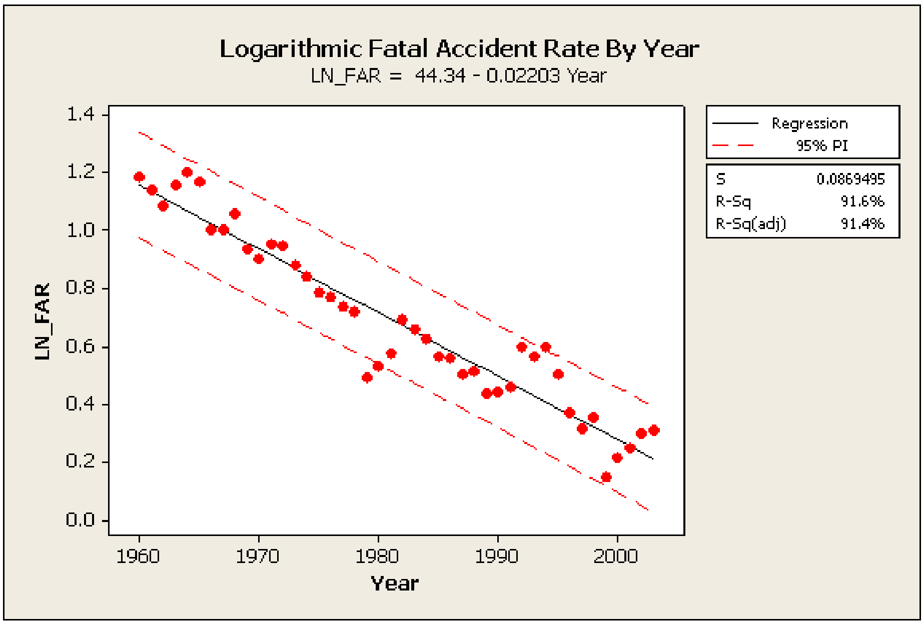

Figure 6: *Logarithmic Fatal Accident Rates 1960 - 2003*

the value of μ, Z_{19} and Z_{20} have the following conditional distributions:

$$Z_i|\mu \sim \mathcal{P}o(\mu t_i), \; i = 19, 20. \tag{3.2}$$

Finally, in order to measure the difference in observed fatal accident rates, we introduce the following test statistic:

$$D_1 = \left| \frac{Z_{19}}{t_{19}} - \frac{Z_{20}}{t_{20}} \right|. \tag{3.3}$$

From Table 3 we observe that $Z_{19} = 719$ and $Z_{20} = 631$. Moreover, $t_{19} = 34.887$ and $t_{19} = 38.641$. Thus, we get that: $D_1 = |2.06 - 1.63| = 0.43$. To determine if this number is significant, we compute the tail probabilities of D_1 e.g., by using Monte Carlo simulations. We find that $\Pr(D_1 > 0.43) \approx 0.003$. Thus, we conclude that the observed drop is indeed significant. That is, the fatal accident rate in 1979 is significantly lower than the fatal accident rate in 1978.

Despite the above calculations, one may still argue that the observed drop from 1978 to 1979 is just a random effect that may happen sooner or later in a long time series. In order to strengthen the conclusion we consider a wider section of the observed time series, ranging from 1978 to 1982. Over these years the average value of the observed fatal accident rates is 1.8325. Thus, we let the parameters of the prior be $a = 1.0$ and $b = 0.5457$. The numbers of fatal accidents in the years $1978, 1979, \ldots, 1982$, are $Z_{19}, Z_{20}, \ldots, Z_{23}$. As above we assume that for given μ the Z_is have the following conditional distributions:

$$Z_i|\mu \sim \mathcal{P}o(\mu t_i), \; i = 19, 20, \ldots, 23. \tag{3.4}$$

Finally we introduce the following test statistic:

$$D_2 = \max_{19 \le i \le 23} \{Z_i/t_i\} - \min_{19 \le i \le 23} \{Z_i/t_i\}. \tag{3.5}$$

Since the maximum fatal accident rate is obtained in 1978 while the minimum fatal accident rate is obtained in 1979, we get that $D_2 = 0.43$. To determine if this number is significant, we compute the tail probabilities of D_2 e.g., by using Monte Carlo simulations. We find that $\Pr(D_2 > 0.43) \approx 0.016$. Thus, we conclude that the observed drop is still significant.

A similar question to the above is whether or not the peak in fatal accident rate around the year 1993 is significant. To analyze this we apply the same methods. We start out by considering the jump from 1991 to 1992, where the fatal accident rates are 1.59 and 1.82 respectively, and define the following test statistic:

$$D_3 = \left| \frac{Z_{32}}{t_{32}} - \frac{Z_{33}}{t_{33}} \right|. \tag{3.6}$$

Inserting the observed values we get that : $D_3 = |1.59 - 1.82| = 0.23$. To determine if this number is significant, we compute the tail probabilities of D_3 e.g., by using Monte Carlo simulations. We find that $\Pr(D_3 > 0.23) \approx 0.052$. Thus, we conclude that the observed drop is in fact *not* significant on a 5% level. That is, the fatal accident rate in 1992 is not significantly higher than the fatal accident rate in 1991.

Since the jump in fatal accident rate from 1991 to 1992 is not significant, there is no hope of getting any significant findings by considering a wider section of the observed time series. In fact by using a test statistic similar to D_2, we get a tail probability as high as 25.44 %, which of course is very far from a significant effect.

4 Conclusions and Further Work

In this paper we have seen that both the total accident rates and the fatal accident rates overall are on a steady decline. We have fitted different regression models to the data. Among these models, the loglinear model gives the best fit, indicating that the rates are flattening out. For the total accident rates it seems like there is an even stronger tendency towards flattening. On the other hand for the fatal accident rates there still appears to be a potential for a future decline.

From a long term perspective we find no indication that the trends are affected significantly by events like deregulation. Still the accident rates around the deregulation point are indeed lower than one could expect. Thus, for a limited time such events may have a positive effect in terms of increased risk awareness.

The proposed method for studying local drops and jumps is fairly sensitive. Thus, while it may look like the jump in fatal accident rates around year 1993 is equally significant as the drop around 1980, this turns out to be false. This underlines the importance of a local analysis in addition to the more standard global analysis.

It should be noted, however, that this study is based on accident counts only. Thus, we do not attempt to diagnose the causes of the accidents in any way. In order to better understand the findings regarding the period around the deregulation we suggest that a more thorough study is carried out where also the causes of the accidents are identified. If this is done, one may also find ways to extend the increased risk awareness effects beyond the short period around a triggering event like deregulation.

In addition to accident counts, it is of interest to include the number of fatalities for each incident. Clearly these numbers carry relevant information about the types of accidents. Thus, in a future study we suggest that these numbers are taken into account as well.

One of the reasons why we are able to draw such strong conclusions, is that the number of general aviation accidents is fairly high. Thus, by the law of large numbers, we get statistically stable results. We have tried to obtain similar results for commercial flights. However, for this type of flights, the number of accidents is much smaller. Thus, apart from a rough trend analysis, it is not possible to identify any nontrivial local effects.

Acknowledgments

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References

[1] AOPA (2004) *U.S. General Aviation Accidents, Fatalities, and Rates 1938-2003*,
http://www.aopa.org/special/newsroom/stats/safety.html

[2] Bruce Landsberg (2005) *General Aviation Accidents 10-Year Trends* AOPA Special Report,
http://www.aopa.org/asf/publications/topics/gatrend.pdf

[3] Boeing (2005) *Statistical Summary of Commercial Jet Airplane Accidents: Worldwide Operations 1959-2003* Boeing Report,
http://www.boeing.com/news/techissues/pdf/statsum.pdf

[4] NTSB (2005) *Aviation accident database* NTSB,
http://www.ntsb.gov/ntsb/query.asp

5 Appendix

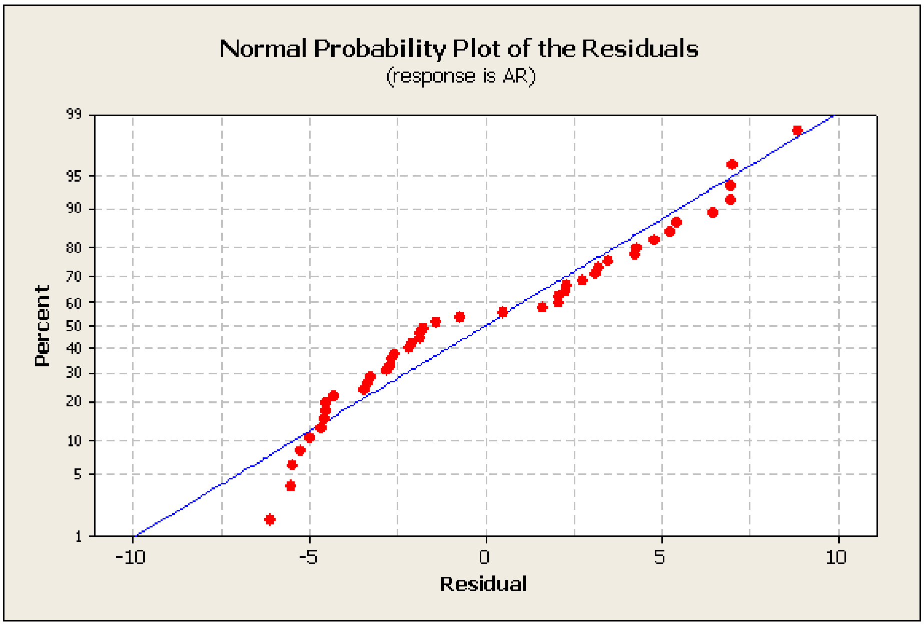

Figure 7: *Normal Plot of Residuals of Total Accident Rates*

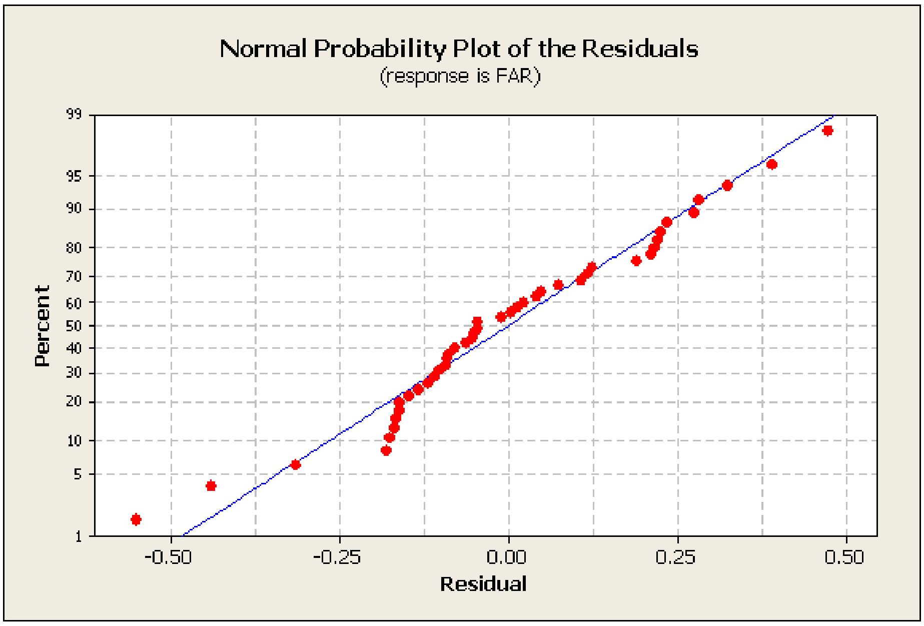

Figure 8: *Normal Plot of Residuals of Fatal Accident Rates*

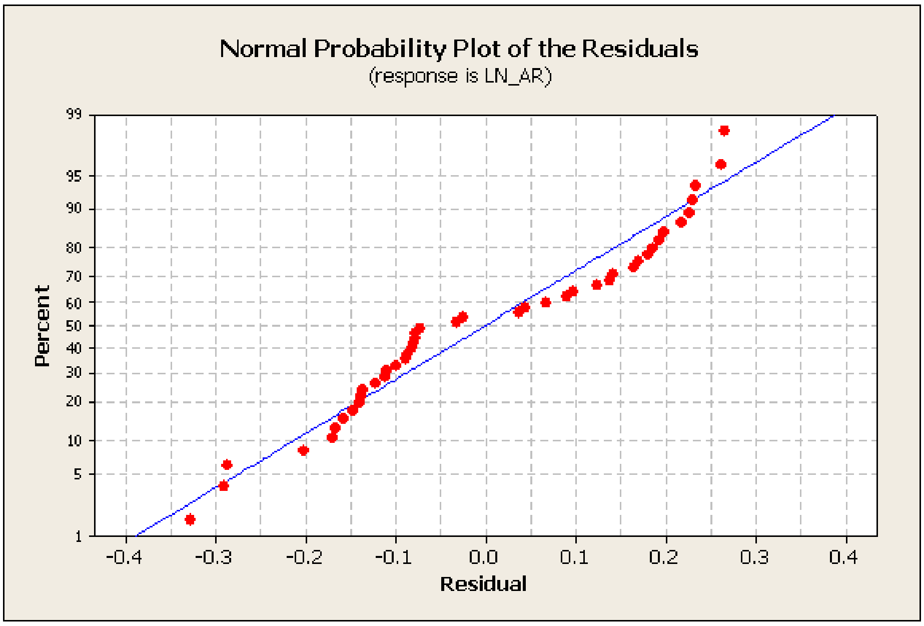

Figure 9: *Normal Plot of Logarithmic Residuals of Total Accident Rates*

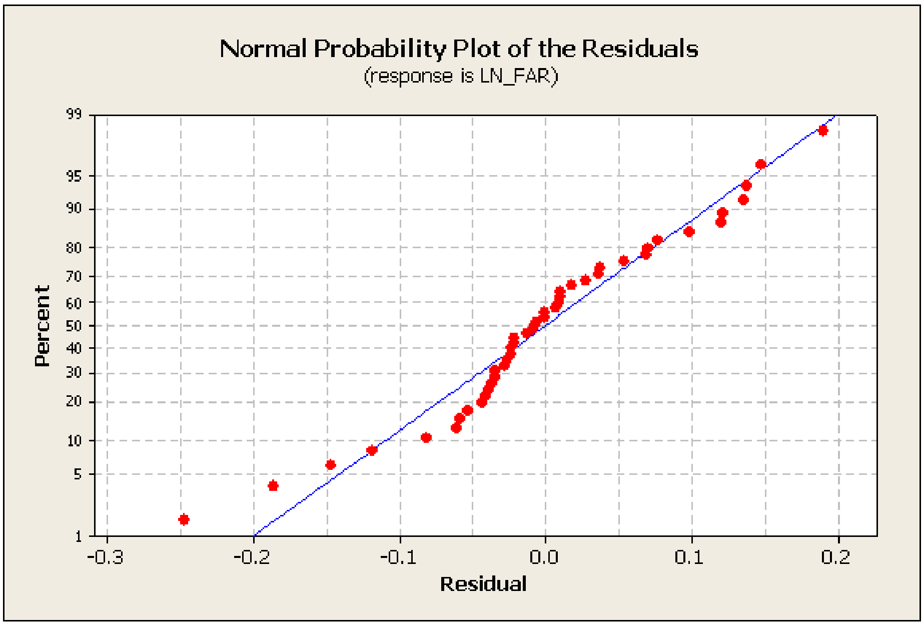

Figure 10: *Normal Plot of Logarithmic Residuals of Fatal Accident Rates*

Year	Tot. Acc.	Fat. Acc.	Fatalities	Hrs. Flown	Acc. Rate	FAR
1960	4,793	429	787	13,121,000	36.53	3.27
1961	4,625	426	761	13,602,000	34.00	3.13
1962	4,840	430	857	14,500,000	33.38	2.97
1963	4,690	482	893	15,106,000	31.05	3.19
1964	5,069	526	1,083	15,738,000	32.21	3.34
1965	5,196	538	1,029	16,733,000	31.05	3.22
1966	5,712	573	1,149	21,023,000	27.17	2.73
1967	6,115	603	1,228	22,153,000	27.60	2.72
1968	4,968	692	1,399	24,053,000	20.65	2.88
1969	4,767	647	1,495	25,351,000	18.80	2.55
1970	4,712	641	1,310	26,030,000	18.10	2.46
1971	4,648	661	1,355	25,512,000	18.22	2.59
1972	4,256	695	1,426	26,974,000	15.78	2.58
1973	4,255	723	1,412	29,974,000	14.20	2.41
1974	4,425	729	1,438	31,413,000	14.09	2.32
1975	3,995	633	1,252	28,799,000	13.87	2.20
1976	4,018	658	1,216	30,476,000	13.18	2.16
1977	4,079	661	1,276	31,578,000	12.92	2.09
1978	4,216	719	1,556	34,887,000	12.09	2.06
1979	3,818	631	1,221	38,641,000	9.88	1.63
1980	3,590	618	1,239	36,402,000	9.86	1.70
1981	3,500	654	1,282	36,803,000	9.51	1.78
1982	3,233	591	1,187	29,640,000	10.91	1.99
1983	3,076	555	1,068	28,673,000	10.73	1.94
1984	3,017	545	1,042	29,099,000	10.37	1.87
1985	2,739	498	956	28,322,000	9.67	1.76
1986	2,581	474	967	27,073,000	9.53	1.75
1987	2,495	446	837	26,972,000	9.25	1.65
1988	2,388	460	797	27,446,000	8.70	1.68
1989	2,242	432	769	27,920,000	8.03	1.55
1990	2,242	444	770	28,510,000	7.86	1.56
1991	2,197	439	800	27,678,000	7.94	1.59
1992	2,111	451	867	24,780,000	8.52	1.82
1993	2,064	401	744	22,796,000	9.05	1.76
1994	2,022	404	730	22,235,000	9.09	1.82
1995	2,056	413	735	24,906,000	8.26	1.66
1996	1,908	361	636	24,881,000	7.67	1.45
1997	1,845	350	631	25,591,000	7.21	1.37
1998	1,904	364	624	25,518,000	7.46	1.43
1999	1,905	340	619	29,246,000	6.51	1.16
2000	1,837	345	596	27,838,000	6.60	1.24
2001	1,726	325	562	25,431,000	6.79	1.28
2002	1,713	345	581	25,545,000	6.71	1.35
2003	1,732	351	626	25,800,000	6.71	1.36

Table 3: U.S. General Aviation Accidents, Fatalities, and Rates - 1960 - 2003